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ABSTRACT

This paper presents a framework utilized in two research projects for mathematics teacher learning based on what is understood about students' mathematics learning. The framework is grounded in a social constuctivist perspective and builds on Karplus' Learning Cycle. A framework for mathematics education identifies a learning cycle that progresses through the following stages: exploration of a mathematical situation, discussion leading to concept identification, and application and extension of new ideas. Labeled as Learning Cycle One, the paper describes how this framework can then be applied to mathematics teacher education in a sequence of five learning cycles. Learning Cycle One, as applied in teacher education, affords preservice or inservice teachers the opportunity to participate as students in mathematics instruction. The teachers' mathematics learning serves as the exploration stage in Learning Cycle Two in which teachers' mathematics learning becomes the object of the exploration, concept identification, and application of Learning Cycle One. Learning Cycle Three sees the same cycle applied to students' learning of mathematics. Learning Cycle Four focuses on learning about the planning of mathematics instruction. Learning Cycle Five focuses on the aspects of teaching mathematics that involve interacting with students in both individual and group situations. The author draws the parallel between learning mathematics and learning to teach mathematics and offers the framework as a structure for thinking about the process of mathematics teacher education. (Contains 11 references.) (MDH)

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LEARNING MATHEMATICS AND LEARNING TO TEACH: LEARNING CYCLES IN MATHEMATICS TEACHER EDUCATION

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LEARNING MATHEMATICS AND LEARNING TO TEACH: LEARNING CYCLES IN MATHEMATICS TEACHER EDUCATION

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Background

The current mathematics reform has included discussion of and inquiry into the nature of mathematics, mathematics learning, and mathematics teaching. A partial consensus has developed with regard to these three areas as represented by the National Council of Teachers of Mathematics standards documents (1989, 1991). The preparation of teachers to carry out mathematics instruction as envisioned necessitates a reexamination of the goals, content, and process of mathematics teacher education. The goals of mathematics teacher education have begun to be articulated (National Council of Teachers of Mathematics, 1991; Committee on the Mathematical Education of Teachers, 1991). The process and content will only be effective to the extent that they fit with the learning processes of prospective and in-service teachers of mathematics.

My colleagues and I have employed two strategies for developing useful conceptions of mathematics teacher learning. The first is building frameworks for mathematics teacher learning based on what we understand about students' mathematics learning. Because mathematics and mathematics teaching are both problem-solving activities, i.e. both predominantly involve solving non-routine problems as opposed to carrying out routine procedures, it seems reasonable that the mechanisms of learning would be parallel. Thus, we can develop useful ways of thinking about teachers' learning to teach, by analogy to students' learning of mathematics. To do so, we must identify appropriate parallels between the two.

The second source of our ideas about learning to be a mathematics teacher is from our empirical studies of mathematics teacher development. Research on teacher development, in light of goals of the current reform, is in its infancy. Our research is based on creating a context that promotes such development[1] in order to be able to study teacher development. The first strategy, considering parallels to mathematics learning, gives us initial frameworks for our experimental interventions with teachers which provide the context for our empirical studies.

This report focuses on the first strategy, the generation of a framework which parallels one which we have developed for thinking about mathematics learning. This

framework has proved useful in two major teacher education research projects: the Educational Leaders in Mathematics Project (Simon & Schifter, 1991) and the Construction of Elementary Mathematics Project (Simon & Blume, 1992). The paper will first describe briefly our framework for mathematics education that has served as the seed crystal for the development of a framework for mathematics teacher education.

The frameworks that I will describe are grounded in a social constructivist perspective on learning and build on Karplus' Learning Cycle (Karplus et al., 1977).

Social Constructivism

The social constructivist perspective (Ernest, 1991; Simon & Schifter, 1991; Cobb, Yackel, & Wood, in press) holds that concurrent with and interacting with the individual's construction of knowledge is a social process of knowledge construction. Individuals involved in learning mathematics are also members of mathematical communities which are involved in the development of shared[2] mathematical perspectives. For the student, the mathematics classroom is such a mathematical community with great impact on their learning. The classroom community is engaged in constituting a shared mathematics as the individuals in the community are involved in developing their individual mathematical understandings. The mathematics of the community constrains the individuals' constructions and the individual constructions of the students constrain the nature of what is socially constituted.

Myths of Constructivism

For clarification, I will discuss a couple of conceptions that I encounter as I talk to colleagues, which do not fit our constructivist view.

Myth 1: A constructivist perspective results in teachers who have no agenda for what students will learn. The view of instruction that I will describe is based on mathematics teachers and teacher educators clearly articulating the important ideas that they see as the goal of their instruction. This is not to be interpreted as "discover my answer." Rather the teacher's role is to create an environment and problematic tasks that are designed to increase the probability that students will generate powerful ideas. Teachers' contributions[3] to the design of the classroom experience is informed by their view of useful mathematical knowledge and their understanding of students' extant knowledge. Tasks which are open to a variety of approaches, which are somewhat ill-defined, and which allow for problem posing, are likely to require more extensive thought and sense making and thus promote deeper and more broadly connected knowledge.

Myth 2: The teaching that some teachers do is "constructivist teaching." Constructivism is a theory of knowledge construction which can be useful in understanding learning. From a constructivist perspective, learning is always a process of construction regardless of the characteristics of the learners context or the presence and nature of instruction. Thus all teaching results in construction. Further, a constructivist view of learning does not translate directly into any particular theory of teaching. Any instructional act that facilitates students' construction of powerful ideas is a useful one. The question then is not "Is the instruction constructivist?" but "Is it effective?" Mathematics educators must build theoretical frameworks that contribute to the effectiveness of teaching. Effective teaching can be defined as teaching which consistently contributes to achieving the goals of the mathematics reform.

The Karplus Learning Cycle

According to Karplus, conceptual learning proceeds from an exploration stage to a concept identification stage and then to an application stage[4] where new ideas are used and extended. The application stage triggers a new level of exploration and the cycle recommences.

The Learning Cycle is based on a constructivist view of learning. In the next section, I will describe how it can be adapted to support, not only individual construction , but also the social construction of knowledge.

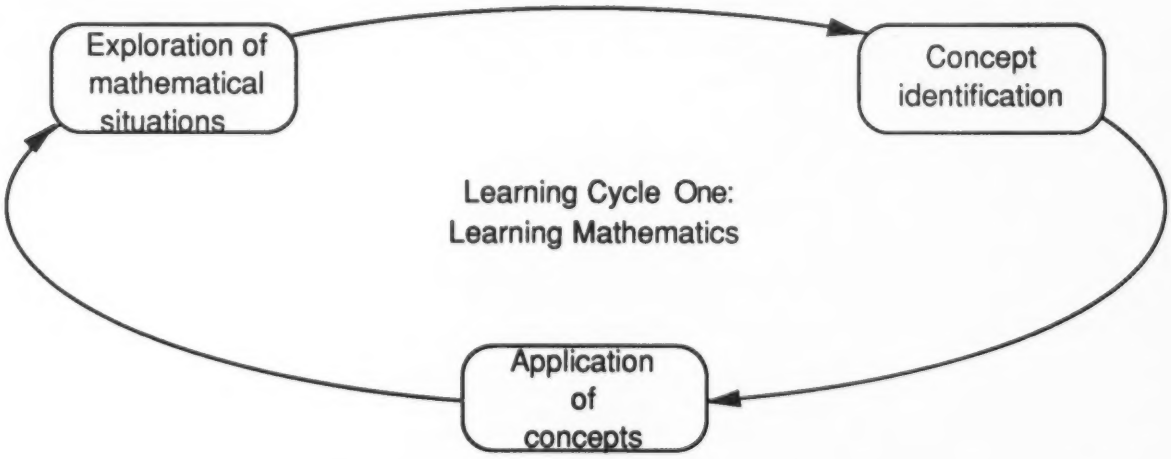
A Framework for Mathematics Education

The need for new frameworks for mathematics education is based on three factors: 1) extant mathematics instruction has not been sufficiently successful in promoting students' development of powerful mathematical ideas and useful conceptions of mathematics; 2) a social constructivist view of mathematics learning seems to suggest that a regimen of teacher showing and telling with students taking notes and practicing what they have been shown is not optimal for supporting the learning processes of students; and 3) if the notion that teachers can "give" students knowledge is abandoned, then we need another approach to instruction. "Let students construct the ideas," is overly simplistic; mathematics instruction must make it possible for students to construct mathematical ideas that have taken the brightest mathematicians thousands of years to develop.

Successful mathematics instruction should result in, not only conceptual development and increased problem solving ability, but also the development of useful conceptions about mathematics. Towards this end, classroom mathematical activity should involve students in exploration of mathematical situations, hypothesis generation, and communication, validation, and application of new ideas. Karplus' Learning Cycle has offered us a useful framework for thinking about mathematics instruction in a way that combines these mathematical activities.

In my mathematics classes, the exploration stage usually involves small groups of students exploring a problem situation. The stage may also involve times when individuals are working alone on the problem (usually outside of class). At a certain point, I call the class together to discuss their work on the problem. This discussion leads into the concept identification stage as key ideas are identified, discussed, debated, revised, and eventually agreed upon. The next phase, which generally involves application and extension of the new ideas, may derive spontaneously from the class discussion or may be triggered by a new problem that I, as the teacher, pose. (See Figure 1.) From a social constructivist perspective, the initial problem situation is designed to demand a level of conceptualization beyond what the students have developed to that point, but not inaccessible to them. When one's prior concepts are inadequate for the task at hand, disequilibrium occurs triggering the learning process.

Figure 1:



In their small groups, students have the opportunity to negotiate shared ideas. This negotiation continues as small groups join together in the whole class discussion.

The teachers' questions promote an abstracting of the ideas which were developed in the context of the problem.

Note that this approach to instruction has some fundamental differences when compared to traditional mathematics teaching. These include:

Proposed Framework	Traditional Instruction
1. explicit planning for communication of mathematical ideas and negotiation of meaning	1. little or no planned communication of mathematical ideas and explicit negotiation of meaning
2. problem solving in a specific context followed by abstraction/ generalization of ideas	2. presentation of an abstract idea followed by application in specific contexts
3. the concepts discussed are developed by the students and expressed in their language	3. the concepts discussed are presented by the teacher in his/her own language
4. the validity of ideas is determined by the classroom community	4. the validity of ideas is determined by the teacher or textbook
5. application is the exploration of new ideas or extensions of ideas previously developed	5. application is the practice in and limited to the use of the general idea presented

Our framework for mathematics teacher education described in the next section is built directly on this framework for mathematics education.

Mathematics Teacher Education

The National Council Teachers of Mathematics (1991) identified five major shifts that need to take place in current instructional practice in order for its goals for students to be met.

We need to shift:

1) toward classrooms as mathematical communities away from classrooms as simply a collection of individuals;

2) toward logic and mathematical evidence as verification away from the teacher as the sole authority for right answers;

3) toward mathematical reasoning away from merely memorizing procedures;

4) toward conjecturing, inventing and problem solving away from an emphasis on mechanistic answer finding; and

5) toward connecting mathematics its ideas and its applications away from treating mathematics as a body of isolated concepts and procedures. (p.3)

To provide instruction which incorporates these shifts, we assert that teachers must develop:

1) knowledge of and about mathematics[5];

2) useful and personally meaningful theories of mathematics learning;

3) knowledge of students' development of mathematical ideas;

4) the ability to plan instruction of this nature; and

5) the ability to interact effectively[6] with students.

These must be key components of an agenda for mathematics teacher education.

To generate an instructional framework, we began by applying our social constructivist perspective and the Karplus Learning Cycle to mathematics teacher education. A social constructivist perspective is as appropriate for thinking about how people learn to be teachers as it is in thinking about how students learn mathematics. The Karplus Cycle gives us a structure for thinking about teachers' learning as well. However, in the case of teacher education, we use it not only to think about particular units of instruction (one lesson or a series of lessons), but also to think about learning which spans a course or longer. The result is five related learning cycles.

Learning Cycles One and Two.

In our projects, we have begun by affording the teachers (preservice and in-service) the opportunity to participate as students in mathematics instruction that is

consistent with the emphases of the mathematics reform and with our framework for mathematics instruction based on the Karplus Cycle outlined in the previous section. This first step contributes to the first two areas of teacher development (1. knowledge of and about mathematics, 2. development of personal theories of learning). Teachers' understanding of mathematics, the nature of mathematics, and how mathematics is learned provides the necessary foundation for them to think about teaching; thus they are appropriate foci for initial teacher education efforts.

For the purpose of teachers developing knowledge of and about mathematics, the learning cycle described, for students in general in the previous section, is appropriate (henceforth referred to as "Cycle One"). However, the goal of involving teachers in constructing personal theories of mathematics learning requires a second level of application of the Learning Cycle. Whereas we have already thought about mathematics instruction as proceeding through the cycle of exploration, concept identification, and application, we can also think about developing an understanding of learning as proceeding through those same steps. Our work with teachers (Simon & Schifter, 1991) demonstrates that teachers can learn a great deal about how mathematics is learned in the context of their own mathematics learning when it is accompanied by reflection on their learning. The teachers' mathematics learning (Learning Cycle 1) serves as the exploration stage for Learning Cycle Two which concerns understanding mathematics learning. In this second learning cycle, teachers' mathematics learning gives them an opportunity to explore mathematics learning in new ways. Their mathematical activity is accompanied by reflection on their mathematics learning through reflective journals and classroom discussion. In our projects, this exploration stage, which incorporates many cycles of mathematics learning, is followed by a concept identification stage in which teachers collectively discuss mathematics learning in an attempt to synthesize important ideas from the diverse experiences that they have had learning mathematics. This is a recursive model (see Figure 2) in which Learning Cycle 1 is embedded in Learning Cycle Two.

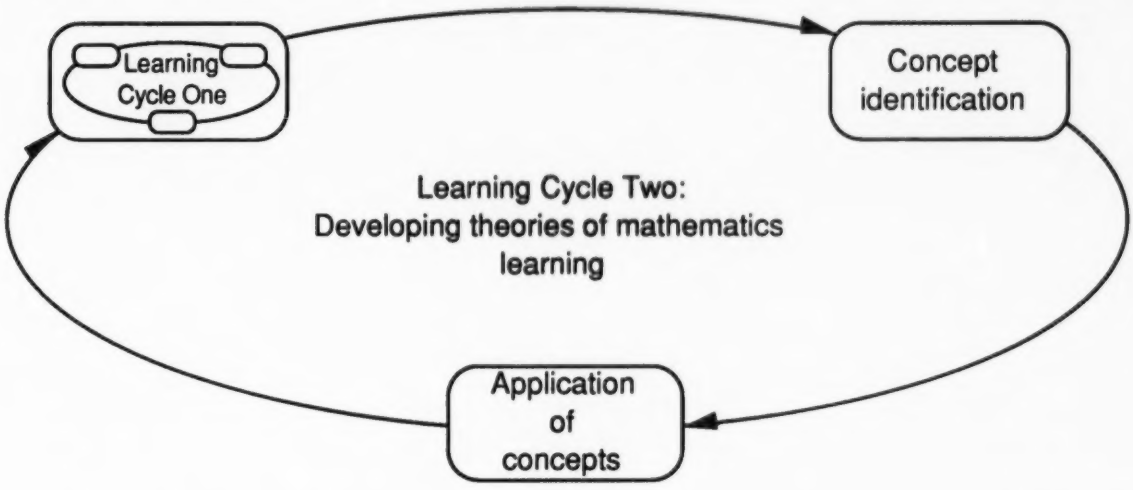
Finally, the teachers have the opportunity to apply their theories of mathematics learning. The application stage of Learning Cycle Two is incorporated into the exploration stage for Learning Cycles Three, Four, and Five.

Learning Cycle Three.

Learning Cycle Three begins with teachers' exploration of students' learning of mathematics. This exploration involves teachers in making sense of interviews with

individual students or pairs of students (initially through observation of live or videotaped sessions, and later through interviews that the teachers conduct themselves)

Figure 2:



and students' mathematical activity in classrooms (again initially through observation and later in classrooms where they are the teachers). This exploration of students' mathematical learning is related to the previous learning cycle on the learning of mathematics in two ways[7]. First, experiences with students' learning of mathematics contribute to teachers' understanding of mathematics learning in general; as such, these experiences also contribute to the exploration stage for Learning Cycle Two.

Second, teachers' developing theories of mathematics learning guide their sense-making with respect to students' mathematical development. In this sense, the exploration of students' mathematical thinking involves the application of teachers' developing theories of mathematics learning (see Figure 3).

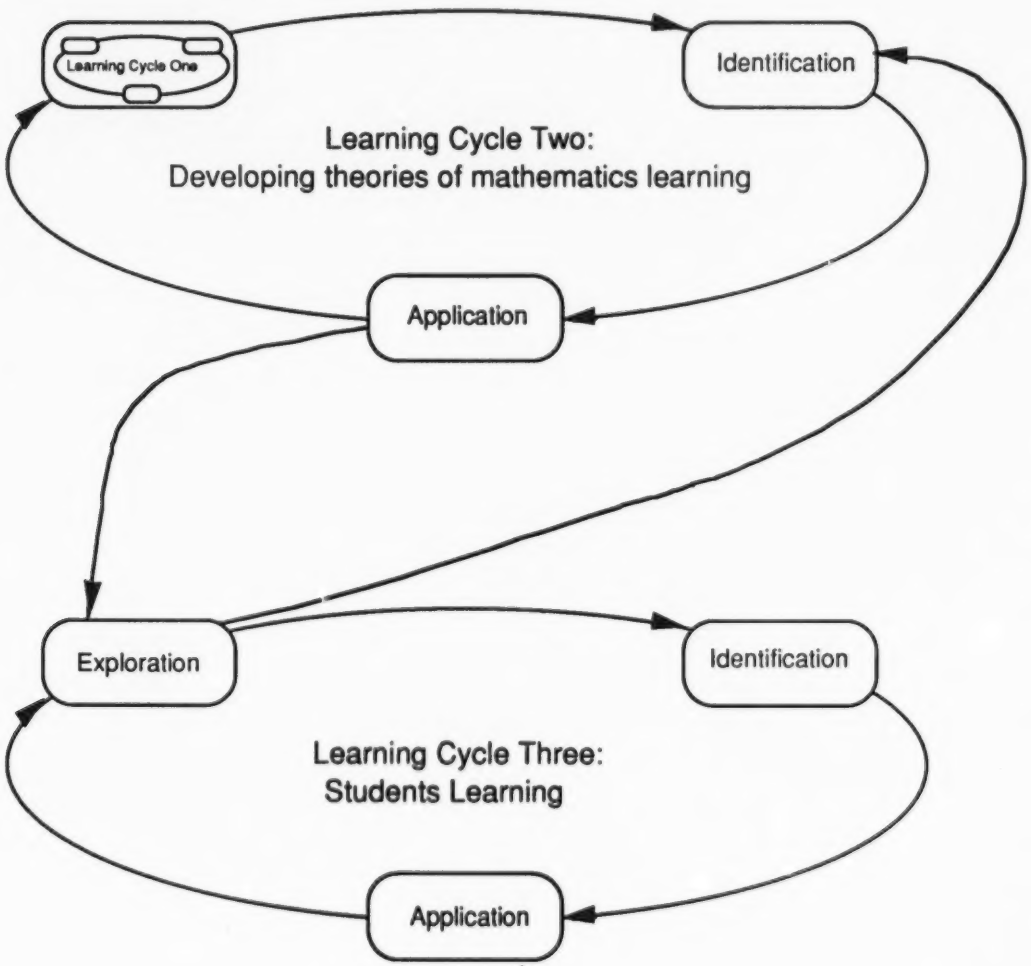
Once again this exploration of students' learning is accompanied by structured reflection which leads into a concept identification phase. Concepts about students' mathematics learning[8] are then applied in Learning Cycles Four and Five which relate to mathematics teaching.

Learning Cycle Four.

The next learning cycle, which focuses on learning about the planning of mathematics instruction, serves as an application phase for each of the first three learning cycles; teachers call upon their mathematical knowledge, their theories of mathematics learning, and their knowledge of students' mathematical development[9]. In

the exploration of planning, teachers initially engage in planning problems or tasks in education classes or workshops where they work with colleagues to develop instructional plans. Later on, instructional planning is directly linked to their teaching practice. Once again reflection on this process is an inherent part of the activity that leads to a concept identification phase in which teachers are articulating for themselves principles of instructional planning. This leads, along with the other learning cycles, into Learning Cycle Five.

Figure 3:



Learning Cycle Five.

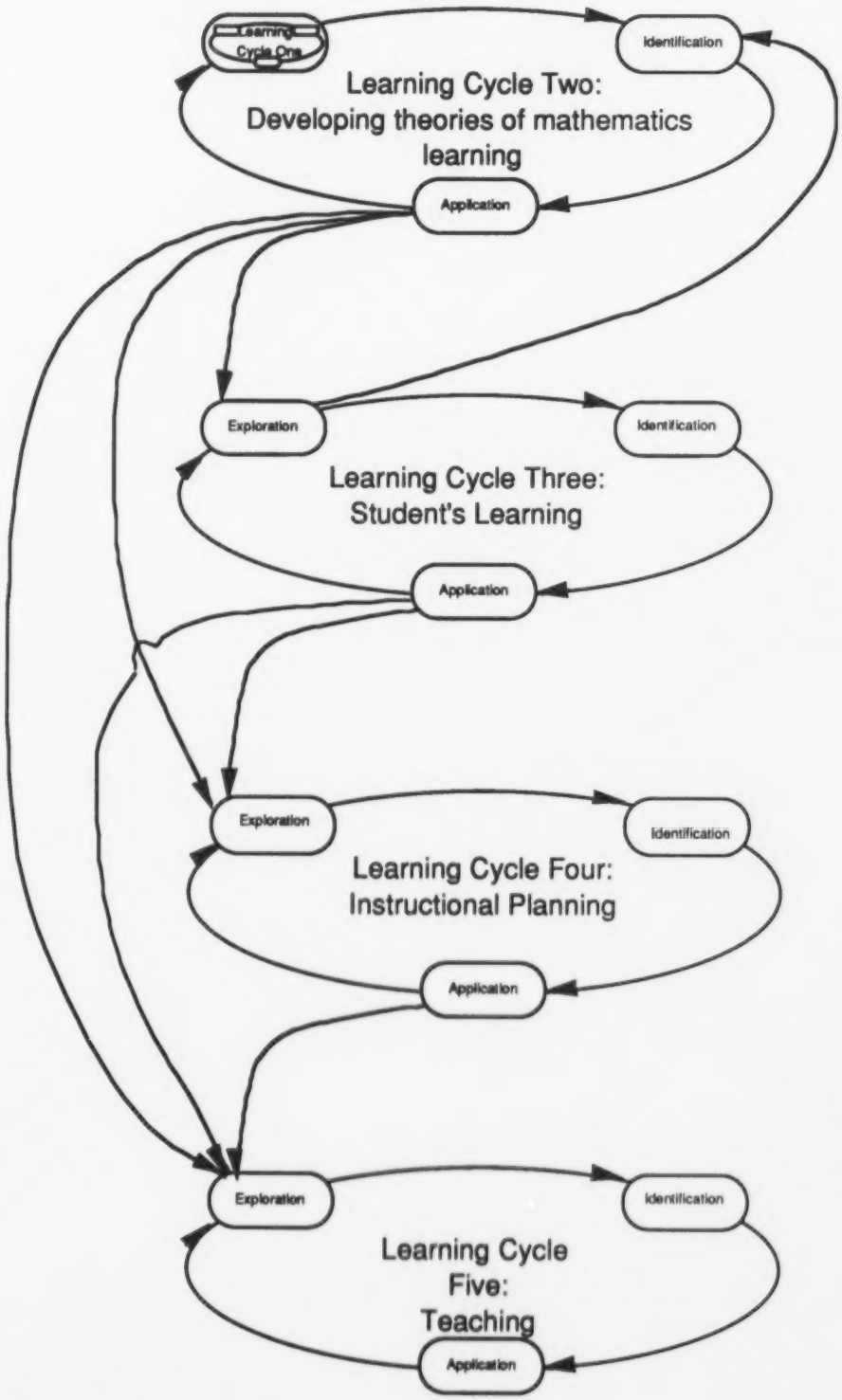
This learning cycle may better be thought of as a set of learning cycles regarding the aspects of teaching that involve interacting with students. The interactions involve listening and questioning in both individual and group situations, facilitating discourse in the classroom mathematics community, and the on-going monitoring of students' learning which results in revising lessons. This exploration takes place almost exclusively in the teachers' own classrooms although such learning is enhanced by viewing colleagues' teaching. Reflection on this exploration takes place individually through reflective journals, in small groups of teachers in support groups, and with clinical supervisors who observe the teachers' as they engage with their students and help them to make sense of the interactions. This reflection leads to concept identification once again. The process of reflection and concept identification may also include large group meetings of teachers involved in a particular program.

Learning Cycle or Web

Although this paper considers only five learning cycles, as with any cycle, there is no end implied here. All of the categories of learning that I have discussed, continue to recycle as life-long learning and professional growth take place.

Whereas Karplus' Learning Cycle is simple to diagram or articulate with its three stages sequenced linearly and temporally, the five learning cycles described are actually related in very complex ways. When one attends to the overlaps and the interconnections[10], one is left with a complex structure that although generated by this simple cycle could best be described as a "web of learning" (see Figure 4). Perhaps it is useful, as it seems to be for our eight-legged friend, the spider, to have organizing principles by which these complex webs are generated.

Figure 4:


Learning Cycle One
Identification
Learning Cycle Two:
Developing theories of mathematics learning
Application
Exploration
Identification
Learning Cycle Three:
Student's Learning
Application
Exploration
Identification
Learning Cycle Four:
Instructional Planning
Application
Exploration
Identification
Learning Cycle Five:
Teaching
Application

Concluding Thoughts

Some mathematics teacher education researchers may find aspects of this theory useful. Certainly others will improve on it. But perhaps more important than the specifics of the theory is that it should serve as a call for the development of theoretical frameworks which can guide research and practice in mathematics teacher education. Again a parallel between mathematics education and mathematics teacher education is in order. In our research, we have observed that it is not so much that teachers must change their theories about mathematics learning as that most teachers have no explicit theory of mathematics learning to guide their teaching. As a result, teaching is shaped by coordinating mandated activities, teaching as teachers have taught in the past, and attempting to incorporate new teaching strategies which are in vogue. Mathematics teacher education seems to suffer from a similar lack of explicit theory with respect to learning to teach mathematics. It is not enough to specify necessary courses, (i.e., methods, content, practicum) nor content or competencies for teacher education programs. We must be guided by theories as to how mathematics teacher educators learn and how teacher education can effectively support these learning processes.

The framework presented not only offers a structure for thinking about the process of mathematics teacher education, it also describes relationships between teachers' mathematics learning, their learning about mathematics learning, and their learning about mathematics teaching. A unique aspect of the framework is the recursive relationship between learning mathematics content and learning to teach mathematics. The framework can also be extended to the issue of learning to be a mathematics teacher educator by moving to the next level of recursion, embedding the learning cycles for mathematics teacher learning into a learning cycle for the learning of mathematics teacher educators. In this case, the learning cycles for mathematics teacher learning would serve as the exploration stage for teacher educators' learning. Just as mathematics teachers learn about the learning of mathematics by being the mathematics students themselves, mathematics teacher educators can learn about the learning to teach mathematics by being the teacher education students themselves and reflecting on this experience.

By including the next level of recursion, we can see how the learning demanded at each level (mathematics students, mathematics teachers, and mathematics teacher educators) increases exponentially. This bears emphasizing since many in the mathematics community do not fully appreciate what it takes for someone who is knowledgeable in mathematics to move to a level of competence as a mathematics

educator as defined by the current reform (National Council of Teachers of Mathematics, 1991). Likewise, many in the mathematics education community do not sufficiently appreciate what it takes for an effective mathematics teacher to become an effective mathematics teacher educator. This framework suggests that the amount of preparation for each level should increase significantly as one moves from mathematics student to mathematics teacher to mathematics teacher educator, an inverse pyramid. In current practice, the pyramid is right-side-up, insufficient mathematics teacher education is provided and little or no preparation is required for those who educate our mathematics teachers.

Footnotes

[1]We use the Cobb and Steffe (1983) constructivist teaching experiment methodology adapted to classroom research as in Cobb, Yackel, and Wood (1988).

[2]From the (constructivist) perspective of the mathematics education researcher, these perspectives are "taken-as-shared" (Cobb, Yackel, & Wood, in press), because individuals have no direct access to the ideas of others.

[3]"Teachers contributions" is meant to underscore that classroom experiences are collectively constituted by teacher and students.

[4]The stages have been labeled differently at different times (although the ideas have remained reasonably constant). We have chosen labels that are useful for us.

[5]Ball (1988) defines knowledge <u>of</u> mathematics as conceptual and procedural knowledge of the subject and knowledge <u>about</u> mathematics as:

> ...understandings about the nature of the discipline--where it comes from, how it changes, and how truth is established. Knowledge about mathematics also includes what it means to "know" and "do" mathematics, the relative centrality of different issues, as well as what is arbitrary or conventional versus what is necessary or logical...

[6]This involves listening, questioning, monitoring, and facilitation of classroom discourse.

[7]To clarify the difference between Learning Cycle Two and Learning Cycle Three: the former is the development of a general theory of learning; the latter is the development of understanding of learning of specific content (e.g. the development of children's counting schemes) and the understanding of the sense-making behaviors of learners at different cognitive and mathematical levels.

[8]Teachers' learning about research on children's learning can play a useful part in either the exploration phase or the extension (application) of new ideasabout students' learning. Carpenter, Fennema, Peterson, and Carey (1988) have made extensive use of introducing primary teachers to research on children's learning.

[9]There are domains of knowledge, that are brought to bear on instructional planning, which we have omitted, such as knowledge of instructional materials

and resources, as well as some domains which span the categories of knowledge which we have generated. Knowledge of mathematical representations is an example of the latter. Knowledge of representations may derive from both teachers' mathematical explorations and their inquiry into students' mathematical development.

[10]The interconnections between cycles go well beyond what has been described and pictured. For example, as teachers focus on students' learning of particular mathematics or on planning instruction, they often explore mathematical ideas in new ways which leads back into Learning Cycle One.

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